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Design of Active Filters Independent of First- and Second-Order Operational Amplifier Time Constant Effects

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Abstract—A practical method of designing active filters in which the transfer function is independent of both first- and second-order operational amplifier time constant effects is presented. Neither matched operational amplifiers nor a tuning procedure dependent on an active parameter is required. The active portion of these filters is universal and readily integrable since it is comprised of conventional operational amplifiers and resistors. The method can be used to design a filter with any realizable transfer function of any order.

Several new filters obtained from this method are introduced and evaluated both theoretically and experimentally. These configurations contain popular passive structures and the new universal active circuits. The significant improvements in filter performance of these new filters is demonstrated in this evaluation.

I. INTRODUCTION

THE response of most active filters employing operational amplifiers (OP AMP's) that are designed to operate at high frequencies and/or high Q's changes significantly if the OP AMP's used in the design are replaced with devices of the same type but with slightly different characteristics. These changes in the response of the active filter are a result of the departure of the magnitude and phase characteristics of the OP AMP's themselves from the ideal values.

During the last few years, research efforts in active filter design have been directed towards topological configurations that are less dependent upon the parameters of the OP AMP's than were previous designs. A host of active filters have appeared in the literature [1]–[7], many of which perform better than the designs of ten years earlier.

Recently several configurations have appeared in the literature in which the authors claimed to have actually eliminated first-order OP AMP time constant effects in some relevant characteristics (e.g., transfer function, pole locations, amplifier gain, etc.) of the filter. These designs can be separated into two disjoint groups based upon the filter performance characteristics.

In one of these groups [8]–[11] the first-order time constant effects are eliminated in the structures proposed by forcing cancellation of the appropriate first derivatives. This cancellation is obtained at the expense of requiring either matched OP AMP's or an active-parameter-dependent tuning procedure. The current practicality of

employing designs requiring matched OP AMP's for proper operation must be addressed since the OP AMP time constants of monolithic OP AMP's are typically matched only to within a few percent due to limitations in the present technology whereas the matching of discrete OP AMP's is generally much worse. Currently designs requiring a cumbersome active-parameter-dependent tuning procedure (which is tantamount to predistortion) don't readily lend themselves to low-cost high-volume production and the performance of such designs is generally adversely affected by changes in temperature or supply voltage as well as aging. This approach has been extended [11] to force the simultaneous cancellation of first- and second-order time constant effects. In these configurations it is even more crucial that OP AMP matching or tuning take place to justify the increased circuit complexity since the magnitude of the uncompensated effects of the second-order terms will typically be small compared to that of the residual first-order terms if perfect cancellation of the first-order derivatives does not take place.

In the other group [1], [3], [7] the first-order time constant effects are eliminated by actually forcing the appropriate first derivatives to vanish. These designs, which are based upon the active sensitivity function [1], are termed zero-active-sensitivity filters and require neither matched OP AMP's nor an active-parameter-dependent tuning procedure.

Geiger and Budak [1] also presented criteria necessary to eliminate second-order as well as first-order time constant effects without requiring matched OP AMP's. No practical filters presented in the literature to date, which require no OP AMP matching or active-parameter-dependent tuning, are independent of both first- and second-order time constant effects.

In this paper a method of designing active filters is introduced in which both the first and second derivatives of the transfer function with respect to the OP AMP time constants vanish. This is achieved by imposing constraints on the gain of the amplifiers to eliminate both first- and second-order time constant effects in any filter employing these amplifiers. Using these constraints the new amplifiers are synthesized. Since the emphasis is placed on the design of the amplifier rather than the filter itself, the ensuing amplifiers are universal and may be used in many filter structures. In addition, these universal amplifiers employ only conventional OP AMP's and resistors and hence are

readily integrable. These amplifiers require neither matched OP AMP's nor an active-parameter-dependent tuning procedure.

Three bandpass configurations employing the proposed amplifiers are introduced. A performance evaluation of these filters confirms the reduced OP AMP dependence of the new designs over that of existing state of the art filters. Experimental results presented agree favorably with the theoretical development of the new filters.

The proposed amplifiers can be used to design active filters with any realizable pole-zero assignment. The results presented for the bandpass situation are representative of the general case.

II. ACTIVE FILTERS WITH ZERO FIRST AND SECOND DERIVATIVES

The voltage transfer function of a general active filter employing a single amplifier with gain $A(s)$, infinite input impedance, zero output impedance, and filter output coincident with that of the amplifier can be expressed functionally as

$$\frac{V_0}{V_i} = T(s) = \frac{-T_1}{T_2 - \frac{1}{A(s)}} \qquad (1)$$

where T_1 and T_2 are transfer functions of a passive RC network.

Assume the $A(s)$ amplifier is constructed from resistors and n ($n \geqslant 1$) internally compensated OP AMP's which are ideal except for a frequency dependent gain given by the expression [12]

$$A_i(s) = \frac{1}{\tau_i s} \qquad (2)$$

where the OP AMP time constant τ_i is the reciprocal of the gain–bandwidth product of the ith OP AMP and is ideally zero. The OP AMP time constants are *not* assumed to be identical.

It follows from a Maclaurin series expansion in the n variables $\tau_1 \cdots \tau_n$ that first- and second-order time constant effects in the *filter transfer function* $T(s)$ are eliminated if $T(s)$ satisfies

$$\frac{\partial T(s)}{\partial \tau_i}\bigg|_{\tau_1 = \tau_2 = \cdots \tau_n = 0} = 0, \qquad \text{for } i \in \{1, 2, \cdots, n\} \quad (3)$$

and

$$\frac{\partial^2 T(s)}{\partial \tau_i \partial \tau_j}\bigg|_{\tau_1 = \tau_2 = \cdots \tau_n = 0} = 0, \qquad \text{for } i, j \in \{1, 2, \cdots, n\}. \quad (4)$$

It should be emphasized that $A(s)$ does not necessarily represent an OP AMP gain and may be ideally either finite or infinite. It should also be emphasized that it is not necessary that the $A(s)$ amplifier be stable by itself since rather the stability of the resulting active filter is of interest.

Under the assumption that all OP AMP's of the filter are internal to the $A(s)$ amplifier, it follows upon taking the

first and second partial derivatives of (1) with respect to the OP AMP time constants that (3) and (4) are satisfied provided

$$\frac{\partial}{\partial \tau_i}\left(\frac{1}{A(s)}\right)\bigg|_{\tau_1 = \tau_2 = \cdots \tau_n = 0} = 0, \qquad i \in \{1, 2, \cdots, n\} \quad (5)$$

and

$$\frac{\partial^2}{\partial \tau_i \partial \tau_j}\left(\frac{1}{A(s)}\right)\bigg|_{\tau_1 = \tau_2 = \cdots \tau_n = 0} = 0, \qquad i, j \in \{1, 2, \cdots, n\}. \quad (6)$$

The realization of $A(s)$ with a minimum number of OP AMP's to satisfy (5) and (6) will now be addressed by considering sequentially the cases $n = 1$, $n = 2$, and $n = 3$.

Case 1: Single OP AMP Internal to the A(s) Amplifier

It was shown in [1] that all first-order pole derivatives with respect to τ can not be forced to vanish in any active RC filter with complex conjugate poles employing a single OP AMP. This result extends readily to imply that

$$\frac{\partial T(s)}{\partial \tau}\bigg|_{\tau=0} \neq 0$$

in any active RC filter with complex-conjugate poles. Thus (5) can not be satisfied with a single OP AMP.

Case 2: Two OP AMP's Internal to the A(s) Amplifier

Examples of amplifiers were given in [7] which satisfied (5). It can, however, be readily shown that it is not possible with just two OP AMP's to design amplifiers with zero first derivatives in which

$$\frac{\partial^2}{\partial \tau_1 \partial \tau_2}\left(\frac{1}{A(s)}\right)\bigg|_{\tau_1 = \tau_2 = 0} = 0.$$

Case 3: Three OP AMP's Internal to the A(s) Amplifier

Assuming the amplifier output coincident with that of one of the OP AMP's it can be shown that all first- and second-order derivatives of $T(s)$ with respect to all OP AMP time constants will vanish provided $A(s)$ is expressible in the form

$$A(s) = \frac{\left[a_0 + s\tau_2 a_2 + s\tau_3 a_3 + s^2\tau_2\tau_3 a_{23}\right]}{\pm\frac{1}{K_0}\left[a_0 + s\tau_2 a_2 + s\tau_3 a_3 + s^2\tau_2\tau_3 a_{23}\right] + s^3\tau_1\tau_2\tau_3} \quad (7)$$

where the a's are real constants with magnitudes less than or equal to unity and $\pm K_0$ is the dc gain of the $A(s)$ amplifier and may be either finite or infinite.

It can be shown that for low frequencies the magnitude and phase functions associated with $A(j\omega)$ given by (7) can be approximated, respectively, by

$$|A(j\omega)| \cong K_0\left[1 \pm \frac{K_0\tau_1\tau_2\tau_3(a_2\tau_2 + a_3\tau_3)\omega^4}{a_0^2}\right]$$

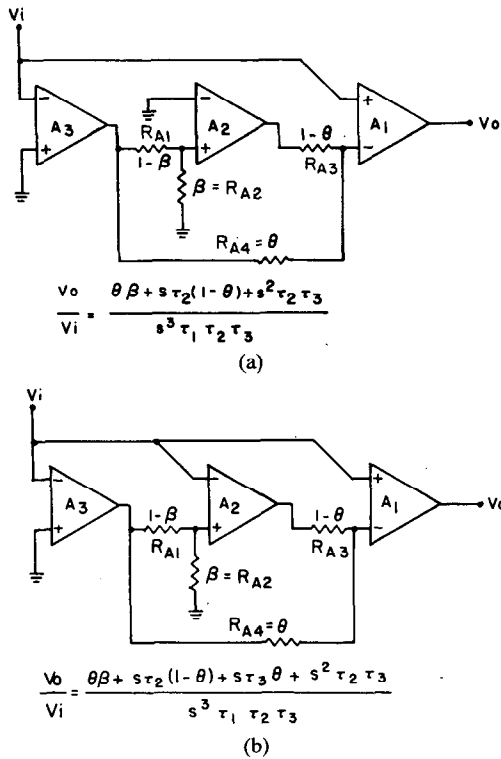$$\theta(\omega) \cong \pm \frac{\tau_1\tau_2\tau_3 K_0\omega^3}{a_0}. \quad (8)$$

$$\frac{V_o}{V_i} = \frac{\theta\beta + s\tau_2(1-\theta) + s^2\tau_2\tau_3}{s^3\tau_1\tau_2\tau_3}$$

(a)

$$\frac{V_o}{V_i} = \frac{\theta\beta + s\tau_2(1-\theta) + s\tau_3\theta + s^2\tau_2\tau_3}{s^3\tau_1\tau_2\tau_3}$$

(b)

Fig. 1. Zero second derivative infinite gain amplifiers.

Note the absence of the ω^2 term in $|A(j\omega)|$ and the ω term in $\theta(\omega)$. Hence, these amplifiers have characteristics that are close to the ideal gain and phase over a wide frequency range.

III. ZERO FIRST AND SECOND DERIVATIVE AMPLIFIERS

Four new amplifiers along with their corresponding gains $A(s)$ are shown in Figs. 1 and 2. Note that these gain expressions agree with the functional form required by (7) to eliminate first- and second-order time constant effects in the transfer function of any filter employing these amplifiers as the active devices. As can be seen, the amplifiers of Fig. 1 have ideally infinite gain whereas those of Fig. 2 ideally realize a finite gain. The infinite gain amplifiers of Fig. 1 are readily integrable and universal in the sense that they may replace conventional OP AMP's in many existing filter configurations. Those in Fig. 1 can be obtained from Fig. 2 by letting $K_0 = \infty$, $r_1 = 1/\theta$, and $r_2 = 1/\beta$. Furthermore, the zero sensitivity property is unaffected if the input leads of any of the OP AMP's are interchanged. For example, reversing the $+$ and $-$ inputs on A_1 in Fig. 1(a) results only in a sign change in front of the transfer function. On the other hand, if the $+$ and $-$ leads on A_1 are reversed in Fig. 2(a), a transfer function with right-half plane poles results thus rendering the finite gain amplifier by itself useless. It should be emphasized, however, that *the overall filter stability criterion* will dictate whether the stable amplifiers of Fig. 2 or the unstable amplifiers obtained by reversing the $+$ and $-$ inputs of any of the OP AMP's in Fig. 2 are to be used in the filter design.

Conspicuously absent from the amplifiers presented in Figs. 1 and 2 are those with a finite negative gain. It can be

$$\frac{V_o}{V_i} = K_0 \frac{1 + s\tau_2\frac{(r_1-1)}{\beta} + s^2\tau_2\tau_3\frac{r_1}{\beta}\frac{K_0}{(K_0-1)}}{1 + s\tau_2\frac{(r_1-1)}{\beta} + s^2\tau_2\tau_3\frac{r_1}{\beta}\frac{K_0}{(K_0-1)} + s^3\tau_1\tau_2\tau_3\frac{r_1}{\beta}\frac{K_0^2}{(K_0-1)}}$$

(a)

$$\frac{V_o}{V_i} = K_0 \frac{1 + s\tau_2(r_1-1)r_2\frac{K_0}{K_0-1} + s\tau_3 r_2\frac{K_0}{K_0-1} + s^2\tau_2\tau_3 r_1 r_2\frac{K_0^2}{(K_0-1)^2}}{1 + s\tau_2(r_1-1)r_2\frac{K_0}{K_0-1} + s\tau_3 r_2\frac{K_0}{K_0-1} + s^2\tau_2\tau_3 r_1 r_2\frac{K_0^2}{(K_0-1)^2} + s^3\tau_1\tau_2\tau_3 r_1 r_2\frac{K_0^3}{(K_0-1)^2}}$$
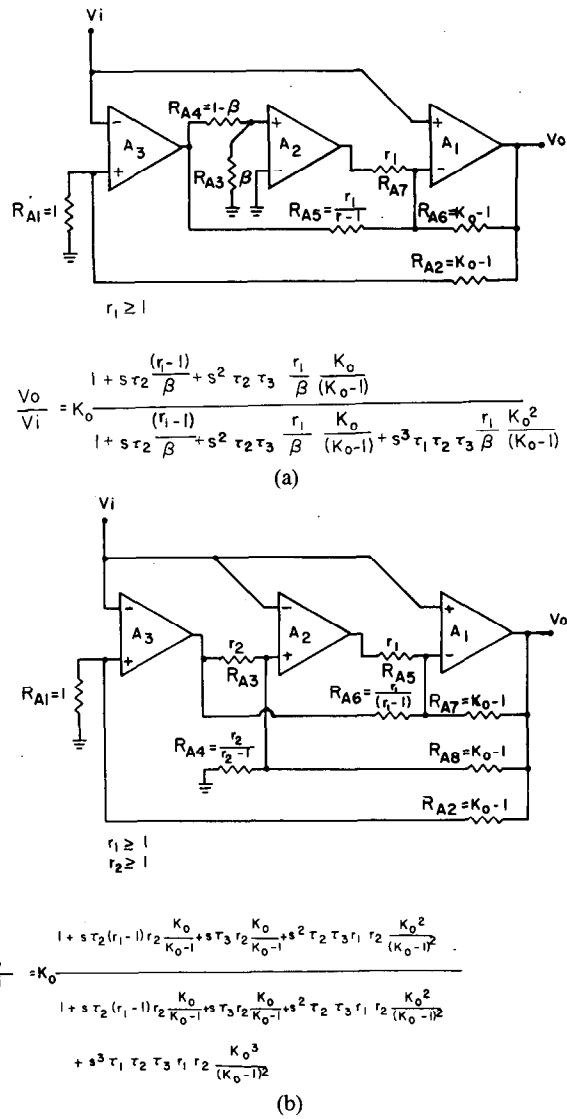
(b)

Fig. 2. Zero second derivative finite gain amplifiers.

readily shown that it is impossible to design an inverting amplifier which satisfies (7) with the infinite input impedance properties enjoyed by the circuits of Figs. 1 and 2 with three OP AMP's.

Consider now the finite input impedance circuit of Fig. 3 which has the well-known transfer function

$$\frac{V_0}{V_i} = -\frac{\dfrac{R_2}{R_1}}{1 + \dfrac{\left[1 + \dfrac{R_2}{R_1}\right]}{A(s)}}. \tag{9}$$

If $A(s)$ is the gain of the amplifier of Fig. 1(a) or 1(b) one readily obtains, respectively,

$$\frac{V_0}{V_i} = -\frac{\dfrac{R_2}{R_1}\left[\theta\beta + s\tau_2(1-\theta) + s^2\tau_2\tau_3\right]}{\left[\theta\beta + s\tau_2(1-\theta) + s^2\tau_2\tau_3\right] + s^3\tau_1\tau_2\tau_3\left(1 + \dfrac{R_2}{R_1}\right)} \tag{10}$$
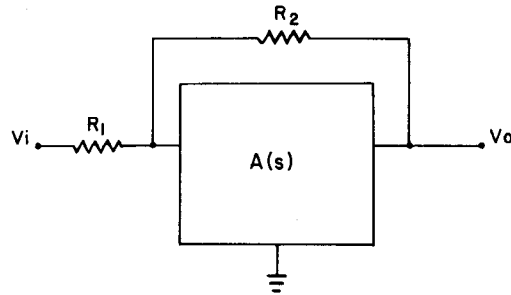
Fig. 3.   General inverting amplifier.

and

$$\frac{V_0}{V_i}$$

$$= -\frac{\frac{R_2}{R_1}\left[\theta\beta+s\tau_2(1-\theta)+s\tau_3\theta+s^2\tau_2\tau_3\right]}{\left[\theta\beta+s\tau_2(1-\theta)+s\tau_3\theta+s^2\tau_2\tau_3\right]+s^3\tau_1\tau_2\tau_3\left(1+\frac{R_2}{R_1}\right)}$$

(11)

which are of the form required by (7) to eliminate both first- and second-order time constant effects in filters employing these amplifiers. It can be seen from (10) and (11) that the ensuing finite input impedance inverting finite gain amplifiers are stable for appropriate choices of θ and β.

Equation (7) will also be satisfied if the amplifiers of Figs. 1(a) or 1(b) with + and − leads of A_1 interchanged are used for the $A(s)$ amplifier in Fig. 3. These inverting finite gain amplifiers are unstable but may well be used in a filter structure to obtain a stable filter.

The amplifier topologies presented above require resistor divider networks to attain the zero first and second derivatives with respect to the τ's. The sensitivities of the ensuing *filter* transfer function to these resistors will now be determined.

If R is a resistor in the $A(s)$ amplifier, it follows that the passive sensitivities are given by

$$S_R^{T(s)}\stackrel{\mathrm{defn}}{=}\frac{\partial T(s)}{\partial R}\frac{R}{T(s)}=\frac{R}{T_{2f}-\frac{1}{A(s)}}\cdot\frac{\partial}{\partial R}\left(\frac{1}{A(s)}\right).$$

(12)

If the ideal amplifier gain is defined by

$$A_{\mathrm{IDEAL}}(s)=A(s)|_{\tau_1=\tau_2=\cdots\tau_n=0}$$

(13)

one can conclude from (12) that the passive sensitivity evaluated at the nominal values $\tau_1=\tau_2=\cdots=\tau_n=0$ is given by

$$S_R^{T(s)}\big|_{\tau_1=\tau_2=\cdots=\tau_n=0}=\frac{-R}{T_{2f}-\frac{1}{A_{\mathrm{IDEAL}}(s)}}\frac{\partial}{\partial R}\left(\frac{1}{A_{\mathrm{IDEAL}}(s)}\right).$$

(14)

It can now be shown that the passive sensitivities evaluated

at the nominal values for the circuits of Fig. 1 are given by

$$S_{R_{Ai}}^{T(s)}=0,\qquad i=1,2,3,4.$$

(15)

For the circuits of Fig. 2 these sensitivities are

$$S_{R_{A1}}^{T(s)}=\frac{\frac{K_0-1}{K_0^2}}{T_{2f}-\frac{1}{K_0}}$$

$$S_{R_{A2}}^{T(s)}=-S_{R_{A1}}^{T(s)}$$

$$S_{R_{Ai}}^{T(s)}=0,\qquad\forall\,i>2.$$

(16)

Thus in the infinite gain case these passive sensitivities vanish and in the finite gain case the nonzero sensitivities are comparable to those obtainable using a single OP AMP in the amplifier.

IV.   SOME NEW FILTERS WITH ZERO TRANSFER FUNCTION SECOND DERIVATIVES

For the purpose of easy comparison with existing designs all new circuits presented in this paper ideally realize a second-order bandpass transfer function. It should be emphasized, however, that the following treatment can be readily modified to synthesize any realizable transfer function of any order. The results given here are to be interpreted as being representative of the general case.

The circuits shown in Fig. 4 are all second-order bandpass configurations in which both the first and second derivatives of $T(s)=V_0/V_i$ with respect to the τ's of the OP AMP's vanish. Many other circuit configurations exist which also have the same property. These three configurations have been chosen for this presentation because their topological structure is similar to that of some well known configurations and many of the passive sensitivity expressions are identical to their well-known counterparts. A discussion of each of these circuits follows.

The circuit of Fig. 4(a) employs the popular bridged-T configuration as the selectivity determining network and uses the infinite gain amplifier of Fig. 1(a) with the + and − leads of A_1 reversed. This configuration is similar to the single-OP AMP circuit employing the bridged-T feedback commonly encountered (see, for example, [13, p. 301] or [14, p. 380]). The sensitivities to the passive parameters of the bridged-T network are the same for both the circuit of Fig. 4(a) and the single OP AMP circuits referenced above.
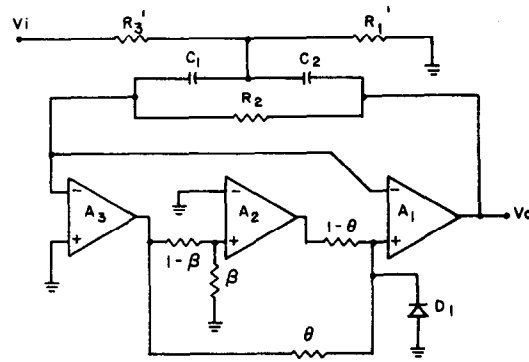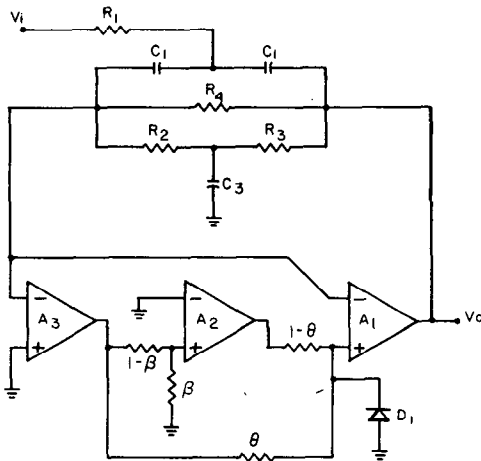
$$A_i = \frac{1}{s\tau_i} \qquad i = 1,2,3$$

$$C_1 = C_2 = C$$

$$R_1 = R_1' /\!/ R_3'$$

(a)

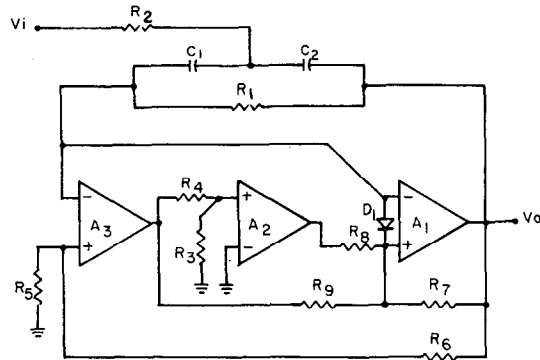$$A_i = \frac{1}{s\tau_i} \quad i=1,2,3 \qquad R_2 = R_3 = R$$

$$C_1 = C_2 = C \qquad\qquad R_1 = R/2$$

$$C_3 = 2C \qquad\qquad R_4 = R/a$$

(b)

$$A_i = \frac{1}{s\tau_i} \qquad i=1,2,3$$

$$h = \frac{R_1}{R_2}$$

$$R_7 = \theta R_5$$

$$R_6 = R_7$$

$$R_3 = \frac{\beta}{1-\beta} R_4$$

$$R_8 = R_9 = 2R_5$$

(c)

Fig. 4. Zero second derivative filters.

Since the infinite gain amplifier of Fig. 1 is used, the passive sensitivities of the filter transfer function to the "θ" and "β" resistors are zero. Although the component spread for this circuit is quite high, it can be reduced considerably by using a "π" to "T" transformation ([15, p. 144]) on the bridging resistor R_2.

The purpose of diode D_1 and the choice of the parameters θ and β are discussed in the following section.

The transfer function of this configuration is

where

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

and

$$Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}.$$

$$T(s) = \frac{V_0}{V_i} = -\frac{s\omega_0\left[\dfrac{2QR_1}{R_3'}\right]\left[1 + s\tau_2\dfrac{(1-\theta)}{\beta\theta} + s^2\tau_2\tau_3\dfrac{1}{\beta\theta}\right]}{\left(s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2\right)\left[1 + s\tau_2\dfrac{(1-\theta)}{\beta\theta} + s^2\tau_2\tau_3\dfrac{1}{\beta\theta}\right] + s^3\tau_1\tau_2\tau_3\dfrac{1}{\beta\theta}\left[s^2 + s\omega_0\left(2Q+\dfrac{1}{Q}\right) + \omega_0^2\right]} \qquad (17)$$

The circuit of Fig. 4(b) employs essentially the same active topology (infinite gain amplifier of Fig. 1(a) with A_1 leads reversed) as the previous circuit but uses the twin-T rather than the bridged-T configuration as the selectivity determining network. For large Q this configuration thus has lower components spread but higher passive sensitivities than the previous circuit. Single-OP AMP filters topologically similar to the circuit of Fig. 4(b) which have identical passive sensitivities with respect to components in the twin-T network can be found in [15, p. 216] or [16, p. 59]. Again, the passive transfer function sensitivities to the resistors in the infinite gain amplifier are all 0.

The transfer function of this configuration is

$$T(s)=\frac{V_0}{V_i}=-\frac{s\omega_0\dfrac{2}{\sqrt{1+2a}}\left[1+s\tau_2\dfrac{(1-\theta)}{\theta\beta}+s^2\tau_2\tau_3\dfrac{1}{\theta\beta}\right]}{\left(s^2+s\dfrac{\omega_0}{Q}+\omega_0^2\right)\left[1+s\tau_2\dfrac{(1-\theta)}{\theta\beta}+s^2\tau_2\tau_3\dfrac{1}{\theta\beta}\right]+s^3\tau_1\tau_2\tau_3\dfrac{1}{\theta\beta}\left[s^2+s\dfrac{\omega_0}{Q}\left(1+\dfrac{2}{a}\right)+\omega_0^2\right]} \tag{18}$$

where

$$a=\frac{1+\sqrt{1+4Q^2}}{4Q^2},\qquad\left(\text{alternatively } Q=\frac{\sqrt{1+2a}}{2a}\right)$$

and

$$\omega_0=\frac{\sqrt{1+2a}}{RC}.$$

The circuit of Fig. 4(c) is similar to the circuits of Deliyannis [17] and Friend [18]. It inherits the passive sensitivities of their configuration and the reduced component spread they obtained by applying positive feedback to the popular infinite-gain bridged-T filter. The passive sensitivities to R_5 and R_6 are identical to those obtained in the Deliyannis–Friend configuration. From (16) it can be seen that the passive sensitivities of the filter transfer function to R_3, R_4, R_7, R_8, and R_9 are all zero. This configuration uses the finite gain amplifier of Fig. 2(a) with the $+$ and $-$ leads of A_1 reversed. Even though the amplifier by itself is unstable, the resulting filter is stable. Of course, the single-OP AMP finite gain amplifier in the filter of Deliyannis and Friend was also unstable by itself. The parameter h can be determined to optimize filter performance in a manner similar to that of Fleischer [5] if desired. The transfer function of this circuit is

and

$$h<4Q^2.$$

V. STABILITY CONSIDERATIONS

The purpose of the small signal diodes and the choice of the parameters θ and β are discussed in this section.

The diodes shown in all three circuits previously discussed are necessary with some commercially available OP AMP's to prevent a large-signal unstable mode of operation of the filters caused by turning on the dc power supplies or by large noise transients. The oscillations are due to nonlinearities of the OP AMP's and have been mentioned occasionally in the literature [2], [3]. Since the diodes conduct only (if at all) during start up of the filter, they need not be included in the analysis.

The choice of the θ and/or β parameters is not arbitrary as it may seem from the expression listed in the previous section for $T(s)$. Although the desired poles are very close to what is expected in the ideal OP AMP case, the parasitic poles introduced by the OP AMP's actually lie in the right half of the s-plane for some values of θ and β. Recently Geiger [19] presented a means for accurately approximating the location of all parasitic poles of active transfer functions provided that $\tau\omega_0\ll1$. It can be readily concluded from Geiger's work that when $\tau\omega_0\ll1$ all parasitic poles of the respective filter transfer functions are in the left half-plane provided that

$$\begin{aligned}\tau_2(1-\theta)&>\tau_1\beta\theta, &&\text{for circuit of Fig. 4(a)}\\ \tau_2(1-\theta)&>\tau_1\beta\theta, &&\text{for circuit of Fig. 4(b)}\\ \tau_2\theta&>\tau_1\beta(1+\theta), &&\text{for circuit of Fig. 4(c)}\end{aligned}\tag{20}$$

Thus if a single-pole model is employed, stability is assured provided θ and β are chosen to satisfy (20). Actually it is necessary in practice to make the inequality a little stronger than indicated by (20) to maintain stability when employing practical OP AMP's such as the 741 or 356. This is

$$T(s)=\frac{V_0}{V_i}=-\frac{s\omega_0\sqrt{h}\,\dfrac{(\theta+1)}{\theta}\left[1+s\tau_2\dfrac{1}{\beta}+s^2\tau_2\tau_3 2\dfrac{(\theta+1)}{\theta\beta}\right]}{\left(s^2+s\dfrac{\omega_0}{Q}+\omega_0^2\right)\left[1+s\tau_2\dfrac{1}{\beta}+s^2\tau_2\tau_3 2\dfrac{(\theta+1)}{\theta\beta}\right]+s^3\tau_1\tau_2\tau_3 2\dfrac{(\theta+1)^2}{\beta\theta^2}\left[s^2+s\omega_0\left(\dfrac{1}{Q}+\dfrac{\sqrt{h}(1+\theta)}{\theta}\right)+\omega_0^2\right]} \tag{19}$$

where

$$\omega_0=\frac{1}{\sqrt{h}\,R_2C}$$

$$\theta=\frac{Qh}{2Q-\sqrt{h}},\qquad\left(\text{alternatively } Q=\frac{\theta\sqrt{h}}{2\theta-h}\right)$$

because the gain of these OP AMP's generally rolls off more rapidly in the 1- to 10-MHz range than the -20 dB/decade predicted by the single-pole model [19]–[22]. A more exact relationship between θ and β to maintain stability obtained by employing a two-pole model of the OP AMP can also be readily obtained from the approximation methods of [19].
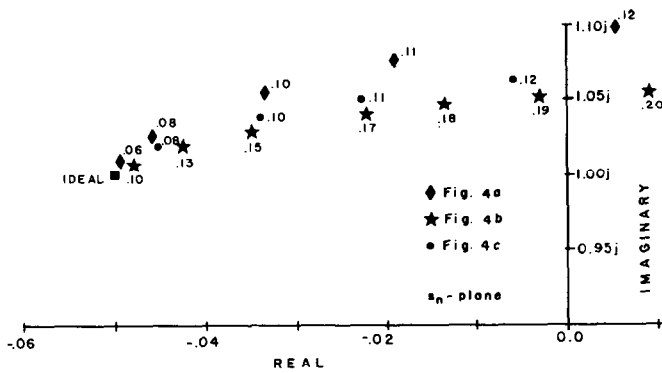
Fig. 5. Pole locus for filters of Fig. 4. τ_n values indicated on plot.

TABLE I

θ AND β VALUES FOR FILTERS OF FIG. 4

Circuit	θ	β
Fig. 4a	.2	1
Fig. 4b	.2	1
Fig. 4c[1]	37.7	.2

1 determined by picking h = 49

VI. FILTER EVALUATION

The circuits of Fig. 4 with θ and β values as given in Table I were all used to realize identical transfer functions (within a magnitude scale factor) with $Q=10$ and $\omega_0=46$ kHz and 62 kHz when the OP AMP's are ideal. In this evaluation all OP AMP's are assumed identical. This assumption is not essential but rather used for convenience. The value of $h=49$ was chosen for convenience in Fig. 4(c). It can be seen that this value of "h" results in a reasonable component spread for $Q=10$.

A pole locus as a function of $\tau_n=\tau\omega_0$ is shown in Fig. 5 for the desired poles of the transfer function. It can be seen that appreciable pole movement is not noticed for $\tau_n \leq 0.05$ for any of the new filters. Thus if an OP AMP with $GB=1$ MHz is employed, filters with center frequencies up to 50 kHz should perform very well. A plot of the transfer function magnitude of these filters appears in Fig. 6.

The merit of the new filters is best established by a comparison with previous state of the art designs. A discussion of the performance of existing designs was presented by Geiger and Budak [1]. A comparison of the pole locus of Fig. 5 and the transfer function magnitude of Fig. 6 with that presented in the previous work readily establishes the merit of the new designs.
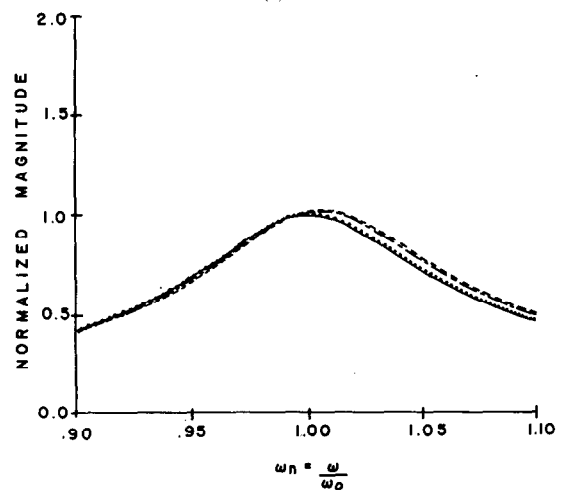
VII. EXPERIMENTAL RESULTS

All three circuits have been analyzed experimentally with favorable agreement of theoretical and experimental results. Details of the experimental performance of the circuit of Fig. 4(c) only are presented here.
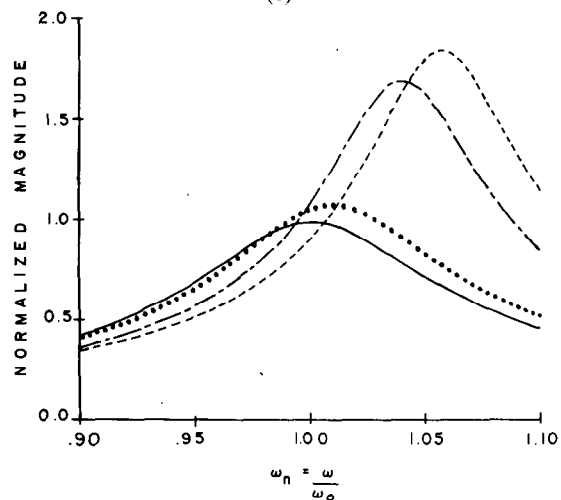
The circuit of Fig. 4(c) was designed to realize bandpass responses with $Q=10$ and center frequencies of $f_0=46$ kHz and $f_0=62$ kHz under the assumption of ideal OP AMP's. The resulting filters were evaluated experimentally using
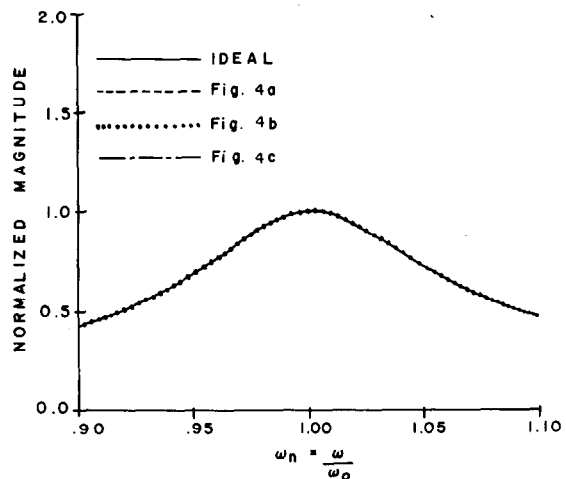
(a)

(b)

$\tau_n = .10$

(c)

Fig. 6. Magnitude response for filters of Fig. 4. (a) $\tau_n=0.025$. (b) $\tau_n=0.05$. (c) $\tau_n=0.10$.

both 741 (measured GB=1.0 MHz±1 percent) and 356 (measured GB=4.15 MHz±10 percent) type of OP AMP's and a 1N4148 diode for D1. A summary of the experimental results along with the passive component values used in this evaluation (as measured on a General Radio 1457

TABLE II
EXPERIMENTAL PERFORMANCE FOR FILTER OF FIG. 4(c)

Passive Components All Tests				
R_1 = 46.97k	R_2 = .929k	R_3 = 208.5 Ω	R_4 = 830.2Ω	R_5 = 474.0 Ω
R_6 = 18.27k	R_7 = 18.36k	R_8 = 1.003k	R_9 = 1.008k	

Test 1 C_1 = 522pf C_2 = 522pf

	Theoretical	Experimental	
OP AMP	Ideal	741	356
f_o	46.128 KHz	46.233 KHz	46.226
% error in f_o		+ .23%	+ .21%
Q	10.23	9.70	10.14
% error in Q		- 5.2%	- .88%
τ_n		.046	.01

Test 2 C_1 = 385pf C_2 = 385pf

	Theoretical	Experimental	
OP AMP	Ideal	741	356
f_o	62.516 KHz	62.635 KHz	62.326 KHz
% error in f_o		+ .19%	- .30%
Q	10.23	9.70	10.075
% error in Q		- 5.2%	- 1.5%
τ_n		.063	.015

digital bridge) are included in Table II.

The close agreement between theoretical and experimental results is obvious from this comparison. The performance of the filter using 741's designed for a center frequency of 62 kHz (τ_n =0.063) emphasizes the usefulness of these filters. Notice also that changing GB by more than 400 percent (i.e., using a 356 rather than a 741) does not significantly affect the filter performance and may well be a better indication of performance than the comparison to the theoretical characteristics which rely on the absolute accuracy of the measured component values as well as the absolute accuracy of the frequency counter used in the experimental evaluation.

The experimental performance at frequencies other than those presented here can be determined quite closely from an examination of the theoretical root locus of Fig. 5 or the transfer function magnitude plots of Fig. 6.

VIII. CONCLUSIONS

It has been shown that it is possible to design active filters with zero first and second transfer function derivatives with respect to the parameters of the OP AMP's. This was attained by first establishing the constraints on the gain of amplifiers necessary to obtain these properties and then synthesizing amplifiers which satisfy the constraints. These new amplifiers are both universal and readily integrable.

Several novel second-order bandpass circuits possessing

these zero first and second transfer function derivative properties have been introduced. They employ popular passive structures and the new amplifiers. These circuits are less dependent upon the parameters of the OP AMP's at low frequencies than previously existing designs. In addition, in the 100-kHz range these circuits perform well using low-cost conventional OP AMP's whereas the performance of most previously existing active filters employing the same operational amplifiers is generally considered inadequate. These improvements in performance are attained without requiring either matched OP AMP's or an active parameter dependent tuning procedure. Experimental results confirmed the predicted performance of the new filters.

The new amplifier can be used to synthesize a filter with any prescribed realizable pole-zero assignment. These filters have the zero first and second transfer function derivative property and offer performance improvements similar to those of the bandpass configurations presented.

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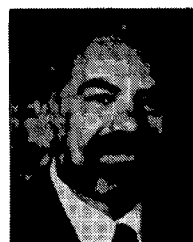
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